Benha University
Faculty of Engineering – Shoubra
Eng. Mathematics & Physics Department
Second Year Mech. Eng. – Power

Final Term Exam
Date: January 9, 2017
Course: Math. and Computer
Duration: 3 hours

- The Exam consists of **Two pages** Answer **All** Questions

No. of questions: 3 Total Mark: 70

Question 1:

25

1) ***Find*** $W^{\backslash}$ **for the function:**

$$W = 2e^{-5z^2} - (2z^{-8}) + \{\cos^2(8z^3)\} - 5^{\sinh 8z}.$$

2) ***Show that*** **the function:**

$$u(x,y) = e^{x}\cdot(\sin y) - y^2 + x^2,$$ **is harmonic,**

and find **the function** $f(z)$; **where**

$$w = f(z) = u(x,y) + i\,v(x,y),$$ **is analytic function,**

And evaluate **the following** $f\left(\frac{-\pi}{3} + \frac{-\pi i}{2}\right)$, $f\left(\frac{-\pi i}{6}\right)$.

3) ***Find*** **the zeros for the following functions:**

i) $f(z) = z^4 - 16i$. ii) $g(z) = \cot an\, z$.

4) ***Using*** **the methods of Residues** ***for evaluate*** **the following integrals:**

i – $\oint_C \left\{\frac{e^{3z}}{(z-i)}\right\} dz$; C is $|z+i| = 3$.

ii – $\oint_C \left\{\frac{\ell n(6+z)}{(z+3)}\right\} dz$; C is $|z-1| = 5$.

iii – $\oint_C \left\{\frac{(4-3z)}{(z-1)\cdot(z^2-3z-4)}\right\} dz$; C is $|z+1| = 4$.

iv – $\oint_C \left\{\frac{\sin(3z)}{(z+2i)^2}\right\} dz$; C is $|z-2i| = 5$.

LOOK ANOTHER PAGE

Question 2: 25

1) ***Find*** **the Fourier series for the function:**

$$f(x) = x^2 \;; \quad x \in (0,\, 2\pi)\,, \quad f(x+2\pi) = f(x)\,.$$

2) ***Write*** **the Fourier integral of the function:** $f(x) = \begin{cases} 2\,; & -1 \le x < 0\,, \\ 4\,; & 0 \le x \le 1\,, \\ 0\,; & |x| > 1\,, \end{cases}$.

3) ***Expand*** **of the function:** $f(x) = x\,; \quad x \in (0,\, \pi/2)\,,$ **period** $= 2\pi$ **,**

***in odd cosine harmonic*.**

4) ***Write*** **the Fourier series of the function:**

$$f(x) = \sin^6 2x\,; \quad x \in [-\pi,\, \pi] \quad \text{and} \quad f(x+2\pi) = f(x)\,.$$

Question 3: 20

1) ***Find*** **the general solution for the following partial differential equations:**

i) $u_{xx} - 6u_{xy} + 9u_{yy} = 12\cos(2x+4y) - 9e^{(8x)}$.

ii) $u_x - 2u_y - u_x = 0\,; \qquad u(x,0) = 4e^{3x}$.

2) ***Put*** **the following complex numbers in another formula:**

i) $z_1 = \{\overline{(-3\,,\sqrt{3}\,)}\}^{-18}$.

ii) $z_2 = \{5 \cdot e^{[-i(29\pi/21)]}\}^7$.

iii) $z_3 = (2\sqrt{3} + 2i)^{-1}$.

iv) $z_4 = \overline{(-1 - i\sqrt{3}\,)} \Big/ \overline{(\sqrt{3} + i\,)}$.

3) ***Find*** **the functions** $u(x,y)\,,\, v(x,y)\,,$ **for the following function:**

$$W = (\overline{3z}) + e^{-(4iz)} - i(\sin \overline{2z}\,)\,.$$

ENDED TEST QUESTIONS

Q, ①

Question (1):.

(1) $W = 2e^{-5Z^2} - (2Z^{-8}) + \cos^2(8Z^3) - 5^{\sinh 8Z}$

$\frac{d}{dz}(2e^{-5Z^2}) = -20Z e^{-5Z^2}$

$\frac{d}{dz}(2Z^{-8}) = -16Z^{-9}$

$\frac{d}{dz}(\cos^2(8Z^3)) = \frac{d}{dz}\left[\frac{1}{2}(1+\cos 16Z^3)\right]$

$= \frac{d}{dz}\left[\frac{1}{2} + \frac{1}{2}\cos 16Z^3\right] = 0 + 24Z^2 \cos 16Z^3$ Sin

$\frac{d}{dz}(5^{\sinh 8Z}) = \frac{d}{dz}(e^{\ln(5)^{\sinh 8z}})$

$= \frac{d}{dz}(e^{\sinh 8Z . \ln(5)}) = 8\cosh 8Z \ln(5)\, e^{\sinh 8Z \ln(5)}$

$\therefore W' = -20Z e^{-5Z^2} + 16Z^{-9} + 24Z^2 \cos(16Z^3)$ Sin

$- 8\cosh 8Z . \ln(5)\, e^{\sinh 8Z . \ln(5)}$

Q1

2

(2) $u_{(x,y)} = e^x \cdot (\sin y) - y^2 + x^2$

$u_x = e^x \cdot (\sin y) + 2x$

$u_{xx} = e^x \cdot (\sin y) + 2$

$u_y = e^x (\cos y) - 2y$

$u_{yy} = -e^x \sin y - 2$

$\therefore u_{xx} + u_{yy} = e^x \cdot (\sin y) + 2 - e^x (\sin y) - 2 = 0$

$\therefore$ the function is harmonic

From Chausy Remans

$u_x = v_y$

$u_y = -v_x$

$\therefore V = \int u_x \, dy \Rightarrow \int [e^x \cdot (\sin y) + 2x] \, dy$

$V = -e^x (\cos y) + 2xy + C(x)$

$V_x = -e^x (\cos y) + 2y + C'(x)$

$\therefore -V_x = u_y$

$\therefore -\cancel{e^x \cdot \cos y} + \cancel{2y} + C'(x) = -\cancel{e^x (\cos y)} + \cancel{2y}$

Q1 (3)

$\therefore C'(x) = 0 \qquad \int C'(x)\, dx = C$

$C(x) = C$

$\therefore V = -e^x (\cos y) + 2xy + C$

$\therefore F(x,y) = U(x,y) + i V(x,y)$
$= e^x (\sin y) - y^2 + x^2 + i[-e^x \cos(y) + 2xy + C]$

$\therefore F(z) = e^z \sin(0) - (0)^2 + z^2 + i[-e^z \cos(0) + 2z(0) + C]$

$\therefore F(z) = z^2 + i[-e^z + C]$

$\Rightarrow F(\frac{-\pi i}{6}) = \cancel{(-\frac{\pi i}{6})^2 + i[-e^{\frac{-\pi i}{6}} + C]}\; e^0(\sin -\frac{\pi i}{6}) - (\frac{\pi i}{6})^2 + i[-e^0 \cos \frac{-\pi i}{6} + 2(0) + C]$

$= \cancel{\frac{\pi^2}{6} + i[-e^{-\frac{\pi}{6}i} + C]}$

$F(\frac{-\pi i}{6}) = -i \sinh\frac{\pi}{6} - \frac{\pi^2}{36} + i[\cosh\frac{\pi}{6} + C]$

$\Rightarrow F(\frac{-\pi}{3} + \frac{-\pi}{2} i) = e^{-\frac{\pi}{3}} \sin(-\frac{\pi}{2} i) - (-\frac{\pi}{2} i)^2 + (-\frac{\pi}{3})^2$
$+ i[-e^{-\frac{\pi}{3}} \cos(-\frac{\pi}{2} i) + 2(-\frac{\pi}{3} \cdot -\frac{\pi}{2} i) + C]$

$\therefore F(-\frac{\pi}{3} + -\frac{\pi}{2} i) = -ie^{-\frac{\pi}{3}} \sinh\frac{\pi}{2} - \frac{\pi^2}{4} + \frac{\pi^2}{9} + i[-e^{-\frac{\pi}{3}} \cosh\frac{\pi}{2} + \frac{\pi^2 i}{3} + C]$

$= -\frac{5\pi^2}{36} + i[e^{-\frac{\pi}{3}} (\sinh\frac{\pi}{2} + \cosh\frac{\pi}{2}) + \frac{\pi^2}{2} i + C]$

Q1 4

3)

(i) $F(z) = z^4 - 16i \Rightarrow z^4 = 16i$

$r = \sqrt{(16)^2 + 0} = 16$

$\theta = \tan^{-1}\frac{16}{0} = \pi/2$

$\therefore z^4 = 16 e^{\frac{\pi}{2} i}$

$\therefore z = (16)^{\frac{1}{4}} e^{i(\frac{\pi}{2} + 2\pi n)/4}$ $n = 0, 1, 2, 3$

$\therefore z = 2 e^{i\frac{(\frac{\pi}{2} + 2\pi n)}{4}}$

at $n=0 \rightarrow z = 2e^{i\frac{(\pi/2)}{4}} = 2e^{i\pi/8} = 2(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8})$

$n=1 \rightarrow z = 2e^{i\frac{5\pi}{2}} \Rightarrow 2(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8})$

$n=2 \rightarrow z = 2e^{i\frac{9\pi}{8}} \Rightarrow 2(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8})$

$n=3 \rightarrow z = 2e^{i\frac{13\pi}{8}} \Rightarrow 2(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8})$

ii) $g(z) = \cot z = \frac{\cos z}{\sin z} = 0$

$\cos z = 0 \Rightarrow \therefore z = \pm(2n-1)\pi/2$

7

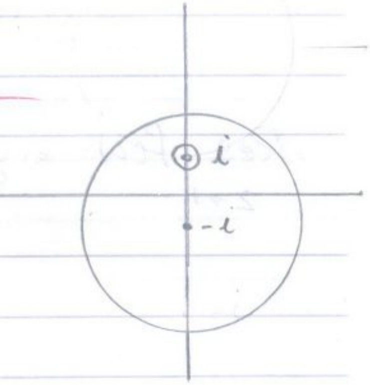
5

(4) i. $\oint_c \left(\frac{e^{3z}}{z-i} \right) dz$ C is $|z+i| = 3$

Poles ⇒ $z = i$ ⇒ inside the Circle

$$\operatorname{Res}_{z \to i} f(z) = \frac{1}{0!} \lim_{z \to i} \left[\cancel{(z-i)} \frac{e^{3z}}{\cancel{(z-i)}} \right] = e^{3i}$$

$$\oint \left(\frac{e^{3z}}{z-i} \right) dz = 2\pi i \cdot e^{3i}$$

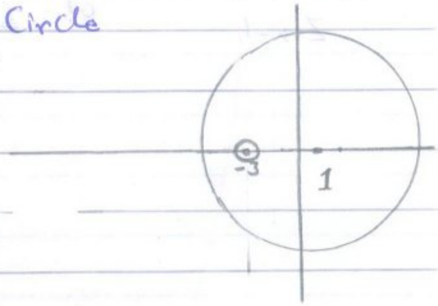


(ii) $\oint \frac{\ln(6+z)}{(z+3)} dz$ C is $|z-1| = 5$

Poles ; $z = -3$ in side the Circle

$$\operatorname{Res}_{z \to -3} f(z) = \frac{1}{0!} \lim_{z \to -3} \left[\cancel{(z+3)} \frac{\ln(6+z)}{\cancel{(z+3)}} \right]$$

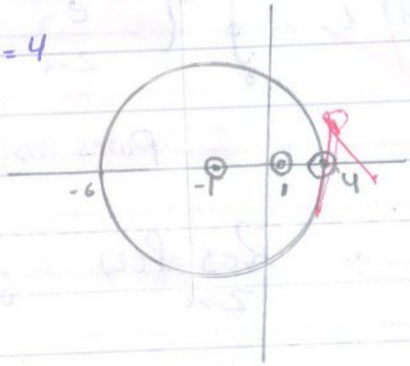
$$= \ln 3$$



$$\therefore \oint \frac{\ln(6+z)}{z+3} dz = 2\pi i \cdot \ln 3$$

Q1 6

(iii) $\oint_c \frac{4-3z}{(z-1)(z^2-3z-4)} \, dz$ C is $|z+1| = 4$



Poles $\Rightarrow z = 1$ & $z = 4$, $z = -1$
inside the Circle

$$\underset{z \to 1}{\text{Res}} f(z) = \frac{1}{0!} \lim_{z \to 1} \left[\cancel{(z-1)} \frac{4-3z}{\cancel{(z-1)}(z-4)(z+1)} \right]$$

$$= \left[\frac{4-3(1)}{(1-4)(1+1)} \right] = -\frac{1}{6}$$

$$\underset{z \to -1}{\text{Res}} f(z) = \frac{1}{0!} \lim_{z \to -1} \left[\cancel{(z+1)} \frac{4-3z}{(z-1)(z-4)\cancel{(z+1)}} \right]$$

$$= \left[\frac{4-(3*-1)}{(-1-4)(-1-1)} \right] = \frac{7}{10}$$

$$\underset{z \to 4}{\text{Res}} f(z) = \frac{1}{0!} \lim_{z \to 4} \left[\cancel{(z-4)} \frac{4-3z}{\cancel{(z-4)}(z-1)(z+1)} \right]$$

$$= \left[\frac{4-(3*4)}{(4-1)(4+1)} \right] = -\frac{8}{15}$$

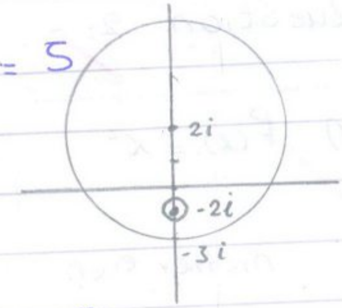
$$\therefore \oint \frac{4-3z}{(z-1)(z^2-3z-4)} \, dz = \left(\frac{-1}{6} - \frac{8}{15} + \frac{7}{10} \right) 2\pi i$$

$$= 0 * 2\pi i = \text{Zero}$$

Q1

7

$$\oint_C \frac{\sin(3z)}{(z+2i)^2}\,dz \qquad C \text{ is } |z-2i| = 5$$



Poles $\Rightarrow z = -2i$

$$\underset{z\to -2i}{\mathrm{Res}}\, f(z) = \frac{1}{1!} \lim_{z\to -2i} \frac{d}{dz}\left[\cancel{(z+2i)^2}\, \frac{\sin(3z)}{\cancel{(z+2i)^2}}\right]$$

$$= \lim_{z\to 2i} 3\cos 3z = 3\cos(3 \times -2i) = 3\cos -6i$$

$$= \underline{3\cosh 6}$$

$$\therefore \oint \frac{\sin(3z)}{(z+2i)^2}\,dz = 2\pi i \times 3\cosh 6 = \underline{6\pi i \cdot \cosh 6}$$

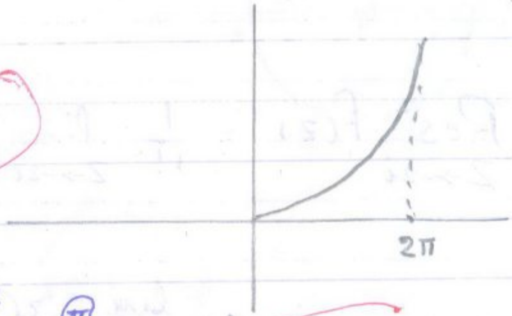
Q2 (8)

Question 2:

1) $f(x) = x^2 \qquad x \in (0, 2\pi) \quad , f(x+2\pi) = f(x)$

niether even nor odd.

$$a_0 = \frac{1}{T}\int_{-T}^{T} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} x^2\,dx$$



$$= \frac{1}{\pi}\left[\frac{1}{2}x^2\right]_0^{2\pi} = \frac{1}{2\pi}\left[2\pi - \pi\right]^2 = \boxed{\frac{\pi}{2}}$$

$2T = 2\pi$

$\boxed{T = \pi}$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \cos nx\,dx$$

x^2	$\cos nx$
$2x$	$\frac{1}{n}\sin nx$
2	$-\frac{1}{n^2}\cos nx$
0	$-\frac{1}{n^3}\sin nx$

$$\Rightarrow \frac{1}{\pi}\left[\frac{x^2}{n}\sin nx + \frac{2x}{n^2}\cos nx - \frac{2}{n^3}\sin nx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{2(2\pi)}{n^2}\cos 2n\pi - 0\right] = \frac{4}{n^2}(-1)^{2n} = \boxed{\frac{4}{n^2}}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \sin nx\,dx$$

x^2	$\sin nx$
$2x$	$-\frac{1}{n}\cos nx$
2	$-\frac{1}{n^2}\sin nx$
0	$\frac{1}{n^3}\cos nx$

$$\Rightarrow \frac{1}{\pi}\left[-\frac{x^2}{n}\cos nx + \frac{2x}{n^2}\sin nx + \frac{2}{n^3}\cos nx\right]_0^{2\pi}$$

2 ~ 9

$$\Rightarrow \frac{1}{\pi}\left[-\frac{(2\pi)^2}{n^2}\cos 2n\pi + \frac{2}{n^3}\cos 2n\pi - \frac{2}{n^3}\cos(0)\right]$$

$$\Rightarrow \frac{1}{\pi}\left[-\frac{4\pi^2}{n^2}(-1)^{2n} + \frac{2}{n^3}\cancel{(-1)^{2n}} - \frac{\cancel{2}}{\cancel{n^3}}\right]$$

$$\therefore b_n = \frac{-4\pi}{n^2}$$

$$\therefore f(x) = \frac{\pi}{2} * \frac{1}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

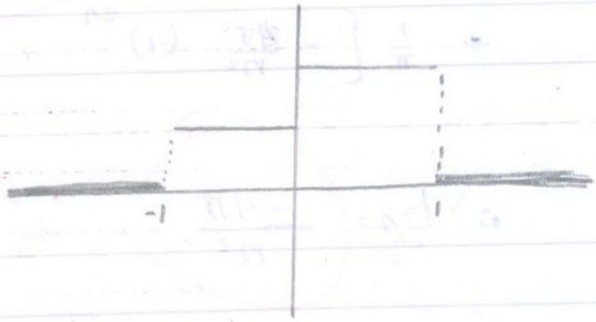
$$= \frac{\pi}{4} + \sum_{n=1}^{\infty}\left(\frac{4}{n^2}\cos nx + \frac{-4\pi}{n^2}\sin nx\right)$$

Q2 (10)

2)

$$F(x) = \begin{cases} 2 & -1 < x < 0 \\ 4 & 0 \leq x \leq 1 \\ 0 & |x| > 1 \end{cases}$$

niether even nor odd.



$$A(\lambda) = \frac{1}{\pi} \int_{-\infty}^{\infty} F(x) \cos \lambda x \, dx$$

$$\Rightarrow 0 + \frac{1}{\pi} \int_{-1}^{0} 2 \cos(\lambda x) \, dx + \frac{1}{\pi} \int_{0}^{1} 4 \cos(\lambda x) \, dx + 0$$

$$= \frac{1}{\pi} \left[\frac{2}{\lambda} \sin(\lambda x) \right]_{-1}^{0} + \frac{1}{\pi} \left[\frac{4}{\lambda} \sin(\lambda x) \right]_{0}^{1}$$

$$A(\lambda) = \left[0 - \frac{2}{\lambda} \sin(-\lambda) \right] + \left[\frac{4}{\lambda} \sin(\lambda) - 0 \right]$$

$$\therefore A(\lambda) = \frac{6}{\lambda} \sin \lambda$$

$$B(\lambda) = 0 + \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \sin(\lambda x) \, dx = \frac{1}{\pi} \int_{-1}^{0} 2 \sin(\lambda x) dx + \frac{1}{\pi} \int_{0}^{1} 4 \sin(\lambda x) \, dx + 0$$

$$\Rightarrow \frac{1}{\pi} \left[-\frac{2}{\lambda} \cos(\lambda x) \right]_{-1}^{0} + \frac{1}{\pi} \left[\frac{-4}{\lambda} \cos(\lambda x) \right]_{0}^{1}$$

$$\therefore B(\lambda) = \left[\frac{-2}{\lambda} - \left(-\frac{2}{\lambda} \cos(\lambda) \right) \right] + \left[\frac{+4}{\lambda} + \left(-\frac{4}{\lambda} \cos(\lambda) \right) \right]$$

$$\therefore B(\lambda) = \frac{2}{\lambda} - \frac{2}{\lambda} \cos(\lambda)$$

$$\therefore I(x) = \frac{1}{\pi} \int_{0}^{\infty} \left(\frac{6}{\lambda} \sin(\lambda) \cos(\lambda x) + \left[\frac{2}{\lambda} - \frac{2}{\lambda} \cos(\lambda) \right] \sin(\lambda x) \right) d\lambda$$

Q2

11

(3) $f(x) = x$ $\quad x \in (0, \pi/2)$ $\quad$ Period: 2π

4

in odd Cosine harmonic

$T = \pi$

$$a_{2n-1} = \frac{4}{T}\int_0^{T/2} f(x) \cos \frac{(2n-1)\pi x}{T}$$

$$a_{2n-1} = \frac{4}{\pi}\int_0^{\pi/2} x \cos(2n-1)x$$

$x \quad \cos(2n-1)x$

$1 \quad \frac{1}{2n-1}\sin(2n-1)x$

$0 \quad \frac{-1}{(2n-1)^2}\cos(2n-1)x$

$$\Rightarrow \frac{4}{\pi}\left[\frac{x}{2n-1}\sin(2n-1)x + \frac{1}{(2n-1)^2}\cos(2n-1)x\right]_0^{\pi/2}$$

$$\Rightarrow \frac{4}{\pi}\left[\frac{\pi}{2(2n-1)}\sin(2n-1)\pi/2 + 0 - \left(0 + \frac{1}{(2n-1)^2}\cos(2n-1)(0)\right)\right]$$

$$a_{2n-1} = \frac{4}{\pi}\left[\frac{\pi}{2(2n-1)}(-1)^{2n-1} - \frac{1}{(2n-1)^2}\right] = \frac{-2}{(2n-1)} - \frac{4}{\pi(2n-1)^2}$$

$(-1)^{(2n-1)} = -1$

$$\therefore f(x) = \sum_{n=1}^{\infty}\left(\frac{-2}{2n-1} - \frac{4}{\pi(2n-1)^2}\right)\cos(2n-1)x$$

Q2 12

(4) $F(x) = \sin^6 2x$

$= (\sin^2 2x)^3 \quad = \left(\frac{1}{2}[1 - \cos 4x]\right)^3$

$= \frac{1}{8}(1 - \cos 4x)^3$

$= \frac{1}{8}(1 + 3\cos^2 4x - 3\cos 4x - \cos^3 4x)$

$= \frac{1}{8}\left(1 + \frac{3}{2}(1 + \cos 8x) - 3\cos 4x - \frac{1}{2}\cos 4x(1 + \cos 8x)\right]$

$= \frac{1}{8}\left[1 + \frac{3}{2} + \frac{3}{2}\cos 8x - 3\cos 4x - \frac{1}{2}\cos 4x\right.$

$\left. - \frac{1}{2}\cos 4x \cos 8x\right]$

$= \frac{1}{8}\left[\frac{5}{2} + \frac{3}{2}\cos 8x - \frac{7}{2}\cos 4x - \frac{1}{4}(\cos(4x - 8x) + \cos(4x + 8x)\right]$

$\Rightarrow \frac{5}{16} + \frac{3}{16}\cos 8x - \frac{7}{16}\cos 4x - \frac{1}{32}\cos 4x + \frac{1}{32}\cos 12x$

$F(x) \Rightarrow \frac{5}{16} + \frac{3}{16}\cos 8x - \frac{15}{32}\cos 4x - \frac{1}{32}\cos 12x$

$\therefore \frac{1}{2}a_0 = \frac{5}{16} \Rightarrow a_0 = \frac{5}{8}$

$a_8 = \frac{3}{16}$ & $a_4 = -\frac{15}{32}$

$a_{12} = \frac{-1}{32}$

Q3 13

Question 3:-

1) (i) $u_{xx} - 6u_{xy} + 9u_{yy} = 12\cos(2x+4y) - 9e^{8x}$

$k^2 - 6k + 9 = 0 \Rightarrow k_1 = k_2 = 3$

$\therefore u_c = f_1(y+3x) + x f_2(y+3x)$

$u_I = \left(\frac{1}{D^2 - 6DE + 9E^2}\right)(12\cos(2x+4y) - 9e^{8x})$

$= \frac{1}{D^2 - 6DE + 9E^2} \cdot 12\cos(2x+4y) - \frac{1}{D^2 - 6DE + 9E^2} \cdot 9e^{8x}$

$D^2 = -m^2$ & $E^2 = -n^2$, $DE = -mn$ | $D = 8$ $\therefore E = 0$

$\Rightarrow \frac{12}{-4 - 6(-8) + 9(-16)}\cos(2x+4y) - \frac{9}{64 - 0 + 0} e^{8x}$

$u_I \Rightarrow -\frac{3}{25}\cos(2x+4y) - \frac{9}{64} e^{8x}$

$u = u_c + u_I$

Q3

14

ii) $U_x - 2U_y - U = 0 \qquad U(x,0) = 4e^{3x}$

let $U = \beta e^{ax+by}$

$U_x = a\beta e^{ax+by}$ & $U_y = b\beta e^{ax+by}$

$\therefore a\beta e^{ax+by} - 2b\beta e^{ax+by} - \beta e^{ax+by} = 0$

$(a-2b-1)\beta e^{ax+by} = 0 \Rightarrow a-2b-1=0$

$\boxed{\therefore a = 2b+1}$

$\therefore U_{(x,y)} = \beta e^{(2b+1)x + by}$

at $U(x,0) \Rightarrow \beta e^{(2b+1)x + b(0)} = 4e^{3x}$

$\therefore \beta = 4$ & $2b+1 = 3 \Rightarrow b = 1$

$\therefore a = 3$

$\therefore U_{(x,y)} = 4e^{3x+y}$

Q3

15

(2) i) $Z_1 = (-3+\sqrt{3}i)^{-18} \Rightarrow (-3-\sqrt{3}i)^{-18}$

$\theta = 2\pi - \tan^{-1}\frac{y}{x}$	$\tan^{-1}\frac{y}{x}$
$\theta = \pi + \tan^{-1}\frac{y}{x}$	$\theta = 2\pi - \tan^{-1}\frac{y}{x}$

$r = \sqrt{9+3} = 2\sqrt{3}$

$\therefore \theta = \pi + \tan^{-1}\frac{\sqrt{3}}{3} = 210^\circ$

$\therefore Z_1 = (2\sqrt{3}\, e^{i\frac{7}{8}\pi})^{-18} = (2\sqrt{3})^{-18}\left[\cos\frac{7}{8}\pi + i\sin\frac{7}{8}\pi\right]^{-18}$

$\therefore Z_1 \Rightarrow (2\sqrt{3})^{-18}\left(\cos\frac{63}{4}\pi - i\sin\frac{63}{4}\pi\right)$

(ii) $Z_2 = (5.\, e^{-i(\frac{29}{21}\pi)})^7$

$\Rightarrow (5)^7 e^{-i\frac{29}{3}\pi} \Rightarrow (5)^7\left[\cos\frac{29}{3}\pi - i\sin\frac{29}{3}\pi\right]$

$\therefore Z_2 = (5)^7\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

(iii) $Z_3 = \frac{1}{(2\sqrt{3}+2i)} * \frac{2\sqrt{3}-2i}{2\sqrt{3}-2i} = \frac{2\sqrt{3}-2i}{12+4} = \frac{\sqrt{3}}{8} - \frac{1}{8}i$

$r = \sqrt{\frac{3}{64} + \frac{1}{64}} = \frac{1}{4}$

$\theta = 2\pi - \tan^{-1}\frac{y}{x} = 2\pi - 30 = 330$

$\therefore Z_3 = \frac{1}{4} e^{i(\frac{11\pi}{6} + 2\pi n)}$ $n=0$

Q3

(16)

(iv)

$$Z_4 = \frac{-1+\sqrt{3}i}{\sqrt{3}-i} \times \frac{\sqrt{3}+i}{\sqrt{3}+i} = \frac{-\sqrt{3}-i+3i-\sqrt{3}}{3+1}$$

$$Z_4 = \frac{1}{4}(-2\sqrt{3}+2i) = -\frac{\sqrt{3}}{2}+\frac{1}{2}i$$

$$\therefore r = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$$

$$\theta = \pi - \tan^{-1}\frac{y}{x} = \pi - 30 = 150$$

$$\therefore Z_4 = e^{i(\frac{5\pi}{6}+2\pi n)} \quad n=0$$

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(3)

$W = 3\bar{z} + e^{-(4iz)} - i\,[\,\sin(2\bar{z})\,]$

$= 3(x-iy) + e^{-4i(x+iy)} - i\sin(2x-2iy)$

$= 3x - 3iy + e^{-4ix}\cdot e^{4y} - i\sin(2x-2iy)$

$= 3x - 3iy + e^{-4ix}\cdot e^{4y} - i\,[\sin 2x\cos(-2iy) + \cos 2x\sin(-2iy)]$

$\Rightarrow 3x - 3iy + e^{-4ix}\cdot e^{4y} - i\,[\sin 2x\cosh 2y - i\cos 2x\sinh 2y]$

$\Rightarrow 3x - 3iy + e^{-4ix}\cdot e^{4y} - i\sin 2x\cosh 4y - \cos 2x\sinh 2y$

$\Rightarrow 3x - 3iy + [\cos 4x - i\sin 4x]\,e^{4y} - i\sin 2x\cosh 4y - \cos 2x\sinh 2y$

$\therefore\ U_{(x,y)} = 3x + e^{4y}\cos 4x - \cos 2x\sinh 2y$

$V_{(x,y)} = -[\,3y + e^{4y}\sin 4x + \sin 2x\cdot\cosh 4y\,]$

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